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Engineering Notes

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Earth Escape Using a Slowly Rotating, Doubly Reflective Solar Sail

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Introduction

A METHOD for escaping planetary orbit using a solar sail was developed in the 1960s by Sands using a double-sided sail rotating about its own axis.[1] Although this method is compatible with very low-authority attitude actuators, the original method depended on several simplifying assumptions such as fixed sun direction and very restricted inclinations. Other work on planetary escape was pursued at that time,[2–4] but interest in planetary-orbiting sails waned until the late 1970s,[5–8] and again until the late 1990s.[9,10] Recently, Coverstone and Prussing presented a technique for solar-sail escape from a geosynchronous transfer orbit (GTO) using a locally optimal energy-increase steering law.[11] The Coverstone–Prussing method is effective for a wide range of initial orbits, and the resulting sail orientation is very similar to Sands's original work. However, their technique requires significant attitude control authority, as the sail must make a rapid realignment maneuver every orbit. Therefore, it was worth examining whether the energy-increase method presented by Coverstone and Prussing could be adapted to Sands's method.

In this Note we demonstrate a technique for Earth escape from any initial orbit using a two-sided, slowly rotating solar sail. The perigee orientation and rotation axis of the sail are selected to maximize the increase in specific energy of the sail over one orbit, and thus the spin rate and spin orientation are free to change throughout the maneuver. It will be shown that these orientations are smooth and continuous throughout the parameter space and that the net energy increase is always positive, allowing the sail to escape from any initial orbit. Simulations are used to demonstrate that the method has consistent behavior over the entire escape phase and to confirm that it has low control torque requirements. The escape performance of this method is compared to the locally optimal energy-increase method.

Rotating-Sail Algorithm

For a two-sided reflecting solar sail at 1 astronomical unit, the acceleration from solar pressure $\boldsymbol{S}$ can be expressed as a function of the unit normal $\boldsymbol{n}$ to one face of the sail (designated the primary face):

$$\boldsymbol{S} = S_0[\mathrm{sgn}(\boldsymbol{n}^T\boldsymbol{\alpha})](\boldsymbol{n}^T\boldsymbol{\alpha})^2\boldsymbol{n} \tag{1}$$

where $\boldsymbol{\alpha}$ is the direction of solar incidence (the sunline) and S_0 is the characteristic acceleration of the sail.

The energy-increase steering law maximizes the instantaneous change in specific orbital energy $\dot{E}$, which is the dot product of the solar pressure acceleration and spacecraft velocity vector $\boldsymbol{v}$:

$$\dot{E} = \boldsymbol{S}^T\boldsymbol{v} = S_0[\mathrm{sgn}(\boldsymbol{n}^T\boldsymbol{\alpha})]\,(\boldsymbol{n}^T\boldsymbol{\alpha})^2\,(\boldsymbol{n}^T\boldsymbol{v}) \tag{2}$$

Because a slowly rotating sail cannot produce a strictly positive value of $\dot{E}$, we choose to maximize the net energy change over one orbit period P:

$$J = \int_0^P \dot{E}\,\mathrm{d}t = S_0\int_0^P [\mathrm{sgn}(\boldsymbol{n}^T\boldsymbol{\alpha})](\boldsymbol{n}^T\boldsymbol{\alpha})^2(\boldsymbol{n}^T\boldsymbol{v})\,\mathrm{d}t \tag{3}$$

The net energy change caused by solar pressure over one orbit is typically three orders of magnitude less than the orbit energy, and thus we assume that the orbital velocity over one period is independent of solar pressure:

$$\boldsymbol{v} = \sqrt{\frac{\mu}{a(1-e^2)}}\begin{bmatrix} -\sin f \\ e+\cos f \\ 0 \end{bmatrix} \tag{4}$$

where f is the true anomaly, a is the semimajor axis, e is the eccentricity, and μ is the Earth's gravitational constant. Given the comparatively short periods of Earth-orbiting spacecraft, we further assume that α is constant over the period of integration. The integral over time can be converted into the integral over eccentric anomaly u:

$$\mathrm{d}t = \frac{r^2}{h}\,\mathrm{d}f = \frac{[a(1-e^2)]^{\frac{3}{2}}}{\sqrt{\mu}(1+e\cos f)^2}\,\mathrm{d}f = \sqrt{\frac{a^3}{\mu}}\,\frac{(1-e^2)}{1+e\cos f}\,\mathrm{d}u \tag{5}$$

where r is the orbit radius and h is the specific angular momentum. The objective function can be transformed to a function of eccentric anomaly using these identities:

$$\cos u = \frac{e+\cos f}{1+e\cos f} \tag{6}$$

$$\sin u = \frac{\sqrt{1-e^2}\,\sin f}{1+e\cos f} \tag{7}$$

Thus, using Eqs. (4–7), and scaling J by (aS_0) to create the nondimensional cost function J^*, Eq. (3) becomes

$$J^* = \frac{J}{aS_0} = \int_0^{2\pi}[\mathrm{sgn}(\boldsymbol{n}^T\boldsymbol{\alpha})](\boldsymbol{n}^T\boldsymbol{\alpha})^2\boldsymbol{n}^T\begin{bmatrix} -\sin u \\ \sqrt{1-e^2}\cos u \\ 0 \end{bmatrix}\mathrm{d}u \tag{8}$$

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Based on the results of Refs. 1 and 11, it is assumed that the sail makes one half-revolution per orbit about an inertially fixed rotation axis $\boldsymbol{q}_0$; this axis is not necessarily normal to the orbit plane. We ensure this rotation rate by defining the sail normal in terms of the orbit's mean anomaly M (which itself is a function of eccentric anomaly, $M = u - e\sin u$):

$$\boldsymbol{n} = \cos(M/2)\boldsymbol{n}_0 + \sin(M/2)\boldsymbol{p}_0 \tag{9}$$

where $\boldsymbol{n}_0$ is the orientation of the primary face at perigee and $\boldsymbol{p}_0$ completes the right-hand set $\{\boldsymbol{n}_0, \boldsymbol{p}_0, \boldsymbol{q}_0\}$. These vectors are expressed by a 3-2-1 Euler rotation from the orbit frame to the sail normal frame using the angles θ_n, ϕ_n, and ψ_n, respectively. The orbit frame O is Earth-centered with the $\hat{O}_1$ axis pointing toward instantaneous perigee, the $\hat{O}_3$ axis normal to the orbit plane, and the $\hat{O}_2$ axis completing the right-hand set. In this frame, the vectors are

$$\boldsymbol{n}_0 = \begin{bmatrix} \cos\theta_n \cos\phi_n \\ \sin\theta_n \cos\phi_n \\ -\sin\phi_n \end{bmatrix} \tag{10}$$

$$\boldsymbol{p}_0 = \begin{bmatrix} \cos\theta_n \sin\phi_n \sin\psi_n - \sin\theta_n \cos\psi_n \\ \sin\theta_n \sin\phi_n \sin\psi_n + \cos\theta_n \cos\psi_n \\ \cos\phi_n \sin\psi_n \end{bmatrix} \tag{11}$$

$$\boldsymbol{q}_0 = \begin{bmatrix} \cos\theta_n \sin\phi_n \cos\psi_n + \sin\theta_n \sin\psi_n \\ \sin\theta_n \sin\phi_n \cos\psi_n - \cos\theta_n \sin\psi_n \\ \cos\phi_n \cos\psi_n \end{bmatrix} \tag{12}$$

Similarly, the incident sunlight can be expressed as a 3-2 Euler rotation using angles θ_s and ϕ_s:

$$\boldsymbol{\alpha} = \begin{bmatrix} \cos\theta_s \cos\phi_s \\ \sin\theta_s \cos\phi_s \\ -\sin\phi_s \end{bmatrix} \tag{13}$$

Thus the sail normal is strictly a function of u, e, and the rotation angles, and we write the objective function as

$$J^*(\theta_s, \phi_s, e, \theta_n, \phi_n, \psi_n) = \int_0^{2\pi} \left\{\mathrm{sgn}\left[\boldsymbol{n}(u, e, \theta_n, \phi_n, \psi_n)^T \boldsymbol{\alpha}(\theta_s, \phi_s)\right]\right\}(\boldsymbol{n}^T\boldsymbol{\alpha})^2 \boldsymbol{n}^T \times \begin{bmatrix} -\sin u \\ \sqrt{1-e^2}\cos u \\ 0 \end{bmatrix} \mathrm{d}u \tag{14}$$

Therefore, given the shape of the orbit e and the relative orientation of sunline ϕ_s, θ_s the optimal sail orientation $\{\theta_n, \phi_n, \text{and } \psi_n\}$ can be determined. For example, Sands's method requires that the sunline be in the orbit plane ($\phi_s = 0$) and the perigee be at local noon ($\theta_s = \pi$). For an initially circular orbit, Sands assumes an initial orientation ($\theta_n = \pi/4$, $\phi_n = 0$, and $\psi_n = 0$), which has corresponding J^* of 2.667; this happens to be the optimal value of J^* for those initial conditions. However, for nonzero values of e, Eq. (14) resists a closed-form solution, requiring numerical optimization.

Numerical Optimization

A quasi-Newton search method was used on Eq. (14) to compute the sail orientation that maximizes J^* across the range of input parameters $\{0 \le e < 1, -\pi/2 \le \phi_s \le \pi/2, -\pi \le \theta_s \le \pi\}$. The optimization space is simplified by three symmetries: the two-sided sail, symmetry about $\phi_s = 0$, and symmetry about $\theta_s = \pi/2$

$$J^*(\theta_s, \phi_s, e, \theta_n, \phi_n, \psi_n) = J^*(\theta_s, \phi_s, e, \theta_n + \pi, -\phi_n, -\psi_n) \tag{15}$$

$$J^*(\theta_s, \phi_s, e, \theta_n, \phi_n, \psi_n) = J^*(\theta_s, -\phi_s, e, \theta_n, -\phi_n, -\psi_n) \tag{16}$$

$$J^*(\theta_s, \phi_s, e, \theta_n, \phi_n, \psi_n) = J^*(\pi - \theta_s, \phi_s, e, 2\pi - \theta_n, -\phi_n, \psi_n) \tag{17}$$

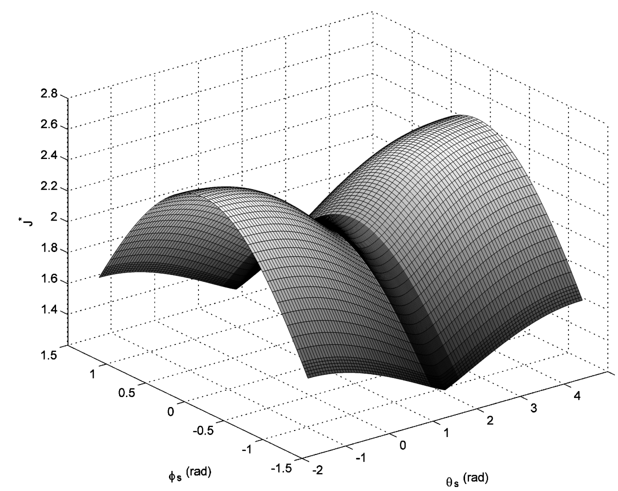

Fig. 1 Optimal energy change J^* as a function of θ_s and ϕ_s for $e = 0.52$.

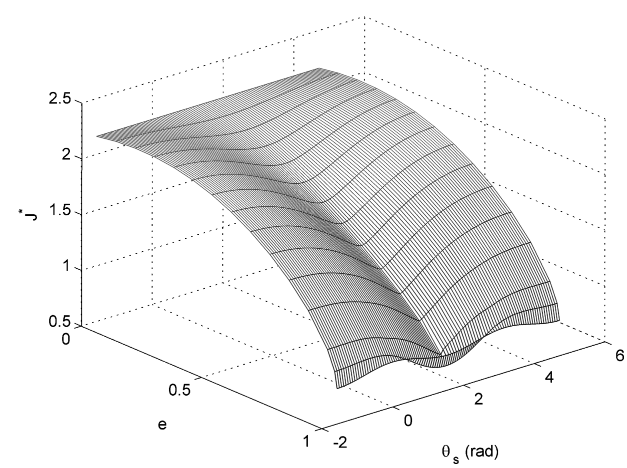

Fig. 2 Optimal energy change J^* as a function of θ_s and e for $\phi_s = 0.3\pi$.

This three-dimensional optimization space is smooth and continuous for all three orientation parameters, and the optimal J^* is always positive. Representative plots of optimal J^* are shown in Figs. 1 and 2; the former is for all solar incidences when $e = 0.52$, and the latter is for all e and θ_s when $\phi_s = 0.3\pi$. These trends hold true throughout the $\{e, \phi_s, \theta_s\}$ space. Thus, it is theoretically possible for a doubly reflective rotating sail to escape from any Earth orbit. (This Note does not consider the situation discussed in Ref. 12, where unfavorable initial conditions lead to a significant increase in eccentricity and drop in the perigee altitude such that the sailcraft crashes.)

Simulation and Results

A three-degree-of-freedom orbit simulator was created, directly integrating the acceleration vector to create the orbital velocity and radius vectors. The governing equation of motion is

$$\ddot{\boldsymbol{r}} = -(\mu/r^3)\boldsymbol{r} + S_0[\mathrm{sgn}(\boldsymbol{n}^T\boldsymbol{\alpha})](\boldsymbol{n}^T\boldsymbol{\alpha})^2\boldsymbol{n} \tag{18}$$

where $\boldsymbol{r}$ is the vector from the center of the Earth to the sailcraft and r is the magnitude of that vector. Earth was assumed to be a perfect sphere, and solar gravitation, lunar gravitation, and atmospheric drag were all ignored, although the shadowing effects of Earth on the orbit were included. In local Earth coordinates, the sunline moved as if the Earth were in a circular orbit around the sun. The simulation was run as a MATLAB®/Simulink application using an Adams solver with absolute and relative tolerances of 10^{-9}. At each time step, Eq. (18) was numerically integrated, the instantaneous radius and velocity vectors were used to compute the instantaneous orbital elements, and the sunline vector was converted into the instantaneous local orbital frame. Next, the optimal sail parameters $\{\theta_n, \phi_n, \text{and } \psi_n\}$ were determined using the lookup table precomputed from Eq. (14);

these values were used in Eqs. (9–11) to compute the sail orientation at that time step.

As the semimajor axis becomes large, the orbit period will be on the order of tens of days, and the underlying assumptions of this method become invalid. Therefore, the locally optimal energy-increase method is substituted for the rotational method once $E > -0.4$ km^2/s^2, corresponding to an orbit with semimajor axis of 80 Earth radii and a rotational rate of $\sim$4 deg/day. In such orbits, the slowly rotating method and the energy-increase method have similar actuator requirements, and the sailcraft typically escapes on that orbit pass.

The performance of this sail is compared against the locally optimal energy-increase method presented in Ref. 11; using the same simulation just outlined the sail normal was instead computed according to their method. A GTO-class orbit was selected, with perigee altitude of 2000 km, an inclination of 0 deg with respect to the ecliptic plane, and a characteristic acceleration of 0.8 mm/s^2. A comparison of the behavior of $\dot{E}$ for both methods is shown in Fig. 3. The locally optimal method always has a nonnegative energy change rate, whereas the rotating sail follows the other method closely except for a small energy loss once per orbit. As shown in Fig. 4, the rotating sail slowly decelerates over the escape trajectory, with deceleration rates on the order of 3 deg/day^2.

In general, escape performance of the rotating sail compares favorably to the energy-increase method. For this Note, "escape" is

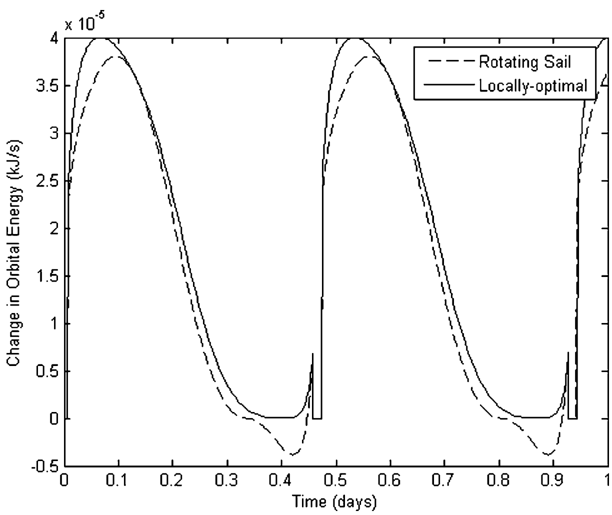

Fig. 3 Instantaneous orbit energy change for rotating-sail and locally optimal methods for first day of GTO escape.

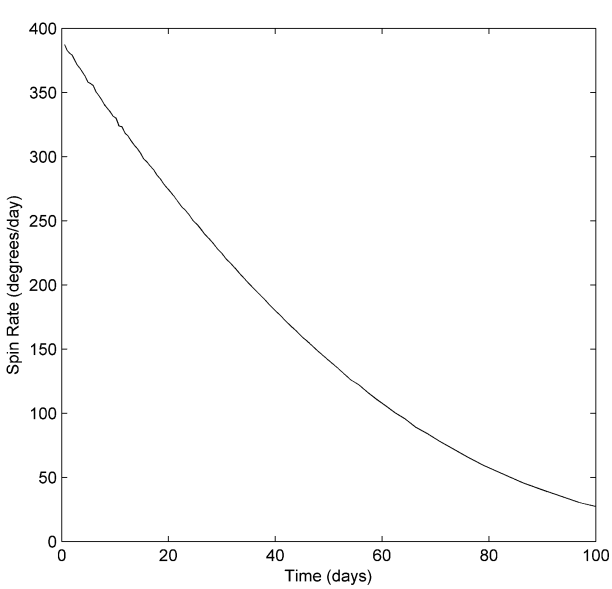

Fig. 4 Spin rate of the rotating sail for the first 100 days of GTO escape.

Table 1 Comparison of escape times and intermediate orbit energies for rotating-sail and locally optimal methods

Inclination to ecliptic, deg	Char. accel., mm/s^2	Escape time, days: Locally optimal	Escape time, days: Two-sided
0	1.0	94	106
10	1.0	94	105
30	1.0	95	104
0	0.8	124	133
10	0.8	133	128
30	0.8	120	131
0	0.6	152	163
10	0.6	150	163
30	0.6	150	167
0	0.4	222	243
10	0.4	224	245
30	0.4	229	267
0	0.2	476	531

defined as the last time that the sail leaves the Earth's sphere of influence (SOI; $r_{\mathrm{SOI}} = 145$ Earth radii), that is, the last SOI crossing before $E \geq 0$. This definition accounts for the fact that a low-thrust sail can reach escape velocity well within the SOI but take days to leave it and the fact that the two-body problem of Eq. (18) becomes invalid as the radius approaches r_{SOI}. The rotating-sail and locally optimal energy-increase methods were compared for a GTO-type mission (2000-km perigee altitude, midday launch, starting at perigee, inclination to the ecliptic varying).

As shown in Table 1, a two-sided, rotating sail has escape times that lag the locally optimal method by 10%. In rare cases, the rotating sail has a shorter escape time; this is because of a favorable sun-sail geometry during the final weeks before escape. In all cases the orbit energy of the rotating sail lags the locally optimal method. A two-sided reflecting sail will have a lower a_0 than an identically sized one-sided sail, owing to the additional mass for the second reflective coating and whatever thermal and structural hardware is needed to accommodate the additional sail mass. However, because a one-sided sail would require additional attitude actuators in order to maintain proper pointing for Earth escape, it is not clear which design would have the better overall a_0.

Conclusions

Sands's original work demonstrated planetary escape for a two-sided, slowly rotating solar sail under a limited set of initial conditions without environmental disturbances. By adapting the locally optimal energy-increase method presented by Coverstone and Prussing, we have extended Sands's concept to an arbitrary initial orbit with escape times comparable to the energy-increase method. Although attitude actuator requirements to maintain the rotating-sail trajectory are quite modest, this Note has not accounted for attitude disturbance rejection, most importantly the significant effect of gravity-gradient torque for near-Earth portions of the orbit. Before feasibility of this method can be assessed, the effects of disturbances such as gravity-gradient torque, Earth oblateness, and residual atmospheric drag must be addressed. The added mass and thermal complexity of a sail with reflective coatings on both sides must also be considered.

The need to compare the escape performance for different low-thrust methods has led to a definition of Earth escape with both orbit energy and orbit radius components. This radius component is necessary because a low-thrust vehicle can achieve escape velocity but still take a nonnegligible time to leave the Earth's sphere of influence; the energy component is necessary because the escape trajectories of low-thrust vehicles often involve very eccentric orbits where the vehicle is far from the Earth but does not possess sufficient energy to escape.

Acknowledgments

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